

Universal Dephasing of Many-Body Rabi Oscillations of Atoms in One-Dimensional Traps

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(Dated: October 15, 2009)

We study a quantum quench in a system of two coupled one-dimensional tubes of interacting atoms. After the quench the system is out of equilibrium and oscillates between the tubes with a frequency determined by microscopic parameters. Despite the high energy at which the system is prepared we find an emergent long time scale responsible for the dephasing of the oscillations and a transition at which this time scale diverges. We show that the universal properties of the dephasing and the transition arise from an infrared orthogonality catastrophe. Furthermore, we show how this universal behavior is realized in a realistic model of fermions with attractive interactions.

PACS numbers: 05.30.Jp, 74.78.Na

Ultracold atomic systems provide a unique laboratory for investigating nonequilibrium dynamics in correlated quantum matter. A common experimental approach is to prepare the system in a simple known quantum state, which is not an eigenstate of the system Hamiltonian, and observe the ensuing time evolution [1, 2, 3]. This is known as a quantum quench. The near perfect isolation of the atoms from the environment allows to concentrate on the intrinsic many-body dynamics following the quench. The focus of this Letter is on the question whether such dynamics can give rise to emergent structures or universal dynamical modes.

The theoretical description of quench phenomena poses a fundamental challenge. Generically, the system is injected with a high energy density. Consequently, the time evolution involves all energy scales. A natural expectation in this case is that the subsequent dynamics would be highly complex and nonuniversal: completely dominated by the microscopic scales at short times; at longer times too complicated to capture in a reasonable theory.

Yet there is accumulating evidence to the contrary. First, emergent phenomena may certainly arise when the energy of the initial state is not too large. In such cases a low energy effective theory appears to give a correct description of the quench and bring out universal aspects of the dynamics. A good example is the decoherence dynamics of a pair of one-dimensional condensates, prepared in a state with well defined relative phase [2, 4]. The dynamics in this case can be understood rather simply within the universal Luttinger liquid description of the condensates [5, 6]. The same analysis holds for the time evolution of coherence between two internal spin states in a one dimensional condensate starting from a perfectly coherent state [3]. Related theoretical questions of quench dynamics within conformal field theories, of which the Luttinger liquid is a particular example, are discussed in Refs. [7, 8, 9].

Second, and more surprisingly, there is numerical evidence for the existence of emergent dynamical modes of very long and possibly diverging time scales, even when

the initial state in the quench is at high energy. Such behavior was seen, for example, in the time evolution of the staggered magnetization in a quantum quench of a spin-1/2 chain [10]. In a wide regime the staggered moment displays rapid oscillations, which clearly reflect the microscopic magnetic exchange interactions. On the other hand, the decay of the oscillations occurs on a much longer time scale, which even diverges on approaching the XY limit of the spin chain. Such behavior is highly suggestive of a theoretical description, which separates out the fast dynamical modes in order to focus on the emergent long time dynamics. We do not know, at present, of a consistent way to accomplish such separation in the spin model of Ref. [10].

In this Letter we achieve separation of scales within a related model of a quantum quench, which is relevant to experiments with one dimensional bosons or fermions in coupled tubes, Fig. 1. Consider a pair of tubes, where in the initial state one of the tubes is at a much lower potential. The ground state corresponds to the quantum liquid filling that tube. How does this ground state evolve when the empty tube is suddenly brought to near resonance with the filled tube?

The regime of interest to us in this work is when the tunnel coupling between the two tubes J , defines the largest frequency scale in the problem. That is, J is much larger than the chemical potential μ . In this regime

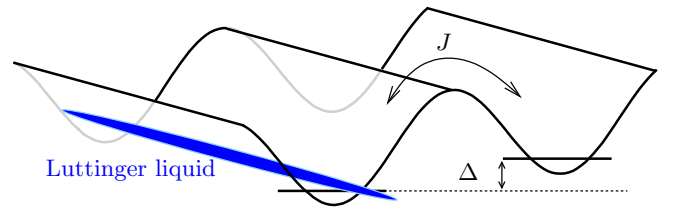


FIG. 1: Two tubes detuned by Δ and coupled via a tunneling amplitude J . For $|\Delta/J| \gg 1$ the ground state is given by all particles residing in the lower tube. We investigate the behavior of the density after a sudden ramp to $\Delta \approx 0$.

the problem defines a direct many-body generalization of the usual Rabi oscillations of a single particle in a double well. The microscopic Hamiltonian of the system is given by $H = H_0 + \sum_{n=R,L} H_n^{\text{ID}}$ with

$$H_0 = \sum_{\sigma} \int \frac{dk}{2\pi} \left(c_{L\sigma k}^{\dagger}, c_{R\sigma k}^{\dagger} \right) \mathcal{H}_0 \begin{pmatrix} c_{L\sigma k} \\ c_{R\sigma k} \end{pmatrix}, \quad (1)$$

$$\mathcal{H}_0 = J\tau^x + \frac{\Delta}{2}\tau^z, \quad (2)$$

$$H_n^{\text{ID}} = \sum_{\sigma} \int \frac{dk}{2\pi} \frac{\hbar^2 k^2}{2m} c_{n\sigma k}^{\dagger} c_{n\sigma k} + g \int dx \rho_{n\uparrow} \rho_{n\downarrow}. \quad (3)$$

Here, τ^{α} are Pauli matrices acting in the pseudo-spin “tube-space” and $n = L, R$ (left/right) is the tube index. Δ is the detuning between the tubes after the quench. The Hamiltonian is written for fermions with mass m and spin $\sigma = \uparrow, \downarrow$; $\rho_{n\sigma}$ is the local density operator. For bosons the Hamiltonian takes an identical form only dropping the spin subscript. The effective contact interaction g depends on the three-dimensional scattering length a_s and the transverse confinement [11, 12].

In absence of interactions, under the influence of H_0 , all particles perform Rabi oscillations of frequency $\Omega = \hbar^{-1} \sqrt{J^2 + \Delta^2/4}$, perpetually in phase with each other. In general, when such rotating spins are coupled to a dynamic environment, the *fast* time-scale Ω^{-1} is supplemented by *slow* time-scales $T_{1(2)}$, responsible for relaxation and dephasing of the Rabi oscillations, respectively. Here, these slow time scales emerge due to an intrinsic mechanism. The fluctuations of the one dimensional quantum liquid will supply the sought slow dynamics in a tractable way.

Before proceeding we outline the general strategy for solving this problem and our main results. In order to separate out the rapid oscillations, we work within a reference frame rotating with the pseudo-spins (interaction picture with respect to H_0). In this frame the two modes corresponding to left and right tubes are replaced by the filled, in-phase rotating mode (mode 1) and the orthogonal, out-of-phase rotating mode (mode 2). Without interactions all particles rotate perpetually in mode 1. The interactions give rise both to an effective detuning Δ_{eff} , as well as particle hopping terms between the two modes. The particle current from the in-phase rotating mode to the out-of-phase mode is the dephasing rate.

For small effective detuning we can compute the dephasing rate by treating the quantum liquid in the filled tube, within its low energy effective theory, the Luttinger liquid. The dephasing rate as a function of effective detuning is found to be a power law $T_2^{-1} \sim (\Delta_{\text{eff}}/\mu)^{\beta}$, where the exponent β can be positive or negative and reflects the critical fluctuations of the Luttinger liquid. A transition to positive β corresponds to an orthogonality catastrophe [13], whereby tunneling of particles out of the Luttinger liquid (dephasing) is suppressed at zero detuning

by the critical fluctuations. For negative β the tunneling (dephasing) rate is enhanced rather than suppressed by the fluctuations. We show that in an experimentally relevant system of fermions with attractive interactions, the exponent β can be tuned from negative to positive by tuning, for example, a_s .

We now turn to derive the main results. The transformation to the rotating frame is accomplished by

$$\begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix} = e^{i\mathcal{H}_0 t/\hbar} \begin{pmatrix} |L\rangle \\ |R\rangle \end{pmatrix}. \quad (4)$$

In this frame H_0 is eliminated and the Hamiltonian is

$$\tilde{H} = \tilde{H}_1 + \tilde{H}_2 + \tilde{H}_{12}(t). \quad (5)$$

The intramode Hamiltonians \tilde{H}_1 and \tilde{H}_2 are of the same form as H_n^{ID} in Eq. (1). The coupling between the two modes $\tilde{H}_{12}(t)$ arises because of the interactions and contains three different contributions: a single particle hopping $\tilde{h}_{1p} \sim ga(t) \sum_{\sigma} \psi_{1\sigma}^{\dagger} \psi_{2\sigma} (\rho_{1-\sigma} - \rho_{2-\sigma}) + \text{H.c.}$; a pair hopping term $\tilde{h}_{2p} \sim gb(t) \psi_{1\uparrow}^{\dagger} \psi_{1\downarrow}^{\dagger} \psi_{2\uparrow} \psi_{2\downarrow} + \text{H.c.}$; and an inter-mode interaction $\tilde{h}_{\text{int}} \sim gc(t) \sum_{\sigma} \rho_{1\sigma} \rho_{2-\sigma} + \psi_{1\sigma}^{\dagger} \psi_{1-\sigma} \psi_{2-\sigma}^{\dagger} \psi_{2\sigma}$. Note that if the interactions were symmetric under $\text{SU}(2)$ rotations in the tube space then \tilde{H}_{12} would vanish. However, the fact that particles interact only within the same tube constitutes a strong breaking of this $\text{SU}(2)$ symmetry.

The coupling constants appearing in $\tilde{H}_{12}(t)$ are time dependent with a periodicity corresponding to a fraction of the Rabi period T . Since the Rabi frequency Ω is the highest frequency scale in the system we can treat this time dependence within a systematic expansion in μ/Ω [14]. This approach is the quantum equivalent of the Kapitza pendulum problem [15]. The leading order contribution is obtained by replacing $a(t)$, $b(t)$, and $c(t)$ by their average value

$$x_{\text{int}} = \frac{1}{T} \int_0^T dt x(t) = \frac{2}{[4 + \alpha^2]^2} \times \begin{cases} \alpha(1 + \alpha^2) & x = a, \\ -2 + \alpha^2 & x = b, \\ -2(1 + \alpha^2) & x = c, \end{cases}$$

where $\alpha = \Delta/J$. The oscillatory time dependence of the interaction leads to sub-leading corrections at order μ/Ω to the calculation of T_2 .

We focus on the effective time independent problem relevant for calculation of T_2 . It consists of a full tube corresponding to the particles in the in-phase mode $|1\rangle$ and an empty tube representing the complementary mode $|2\rangle$. For the purpose of computing the tunneling rate, the empty mode can be represented by

$$\tilde{H}_2 = \sum_{\sigma} \int \frac{dk}{2\pi} \left(\frac{\hbar^2 k^2}{2m} - \mu_2^{\text{eff}} \right) c_{2\sigma k}^{\dagger} c_{2\sigma k}. \quad (6)$$

Here $\mu_2^{\text{eff}} = c_{\text{int}} g \rho_0 / 2$ is a Hartree shift due to the effective interaction between particles in the two modes.

The full mode with density ρ_0 experiences an interaction induced shift in its chemical potential μ_1^{eff} as well. We discuss the exact form of this shift away from the Fermi energy $\epsilon_F = \pi^2 \hbar^2 \rho_0^2 / 8m$ below. Within a microscopic model we will also show that tuning the interactions can lead to an effective resonance between the two modes. Namely $\Delta_{\text{eff}} \equiv \mu_2^{\text{eff}} - \mu_1^{\text{eff}} = 0$.

For the formulation of the long-wavelength Luttinger liquid theory near the effective resonance we introduce the slow fermion modes ψ_{\pm} , such that $\psi_{1\sigma}(x) \approx \psi_{+,\sigma}(x)e^{ik_F x} + \psi_{-,\sigma}(x)e^{-ik_F x}$, and k_F is the Fermi momentum. As usual we express the Hamiltonian (5) using these fields and neglect terms which oscillate as $e^{in k_F x}$. In particular, for this reason the single particle term $\propto a_{\text{int}}$ disappears from the long-wavelength theory.

Next, we bosonize the slow fields via $\psi_{\pm,\sigma}(x) \propto \exp\{-i[\pm\theta_{\rho} - \varphi_{\rho} + \sigma(\pm\theta_{\sigma} - \varphi_{\sigma})]/\sqrt{2}\}$ [16]. We shall concentrate on the case of attractive interactions, where fermions form either (singlet) Cooper pairs or tightly bound molecules [17]. From the point of view of the long-wavelength theory, the fermions form a Luther-Emery liquid with a spin gap Δ_{σ} [18]. The low energy degrees of freedom therefore lie in the charge sector.

$$\tilde{H}_1^{\rho} = \frac{\hbar v_{\rho}}{2\pi} \int dx \left\{ K_{\rho} [\partial_x \varphi_{\rho}(x)]^2 + \frac{1}{K_{\rho}} [\partial_x \theta_{\rho}(x)]^2 \right\}. \quad (7)$$

The phonon velocity v_{ρ} is related to the Fermi velocity $v_F = \pi \hbar \rho_0 / 2m$ by $K_{\rho} = v_F / v_{\rho}$; later on we derive an explicit expression for K_{ρ} from the microscopic theory. The bosonized form of the tunneling terms between the two modes is given by

$$\begin{aligned} \tilde{H}_{12} = & -2g \int dx \left(c_{\text{int}} \left[\cos(\sqrt{2}\theta_{\sigma}) e^{i\sqrt{2}\varphi_{\sigma}} \psi_{2\downarrow}^{\dagger} \psi_{2\uparrow} \right] \right. \\ & \left. + b_{\text{int}} \left[\cos(\sqrt{2}\theta_{\sigma}) e^{-i\sqrt{2}\varphi_{\rho}} \psi_{2\downarrow} \psi_{2\uparrow} \right] + \text{H.c.} \right). \quad (8) \end{aligned}$$

The set of Eqs. (6)–(8) allows us to calculate the evolution of the Rabi oscillations after the quench. We remark that due to the spin gap Δ_{σ} , the first term in (8) is irrelevant for $\Delta_{\text{eff}} < \Delta_{\sigma}$. In the second term of (8), the spin-part of the coupling leads to a logarithmic correction of the “coupling constant” gb_{int} [16]. Since we are only interested in the leading behavior of the dephasing rate, we can focus purely on the charge sector. To second order in the coupling gb_{int} , we obtain via a Fermi golden rule calculation (FGR) the dephasing rate

$$1/T_2^F \sim \int dk dq d\omega d\nu A_2(k, \omega) A_2(q, \nu) \times A_1(-k - q, -\omega - \nu). \quad (9)$$

The spectral densities entering the FGR expression are

$$A_2(k, \omega) = 2\pi \delta(\omega - \hbar^2 k^2 / 2m - \mu), \quad (10)$$

$$A_1(k, \omega) = \frac{1}{\epsilon_F} \left(\frac{\epsilon_F^2}{\omega^2 - \hbar^2 v_{\rho}^2 k^2} \right)^{1 - \frac{1}{2K_{\rho}}}. \quad (11)$$

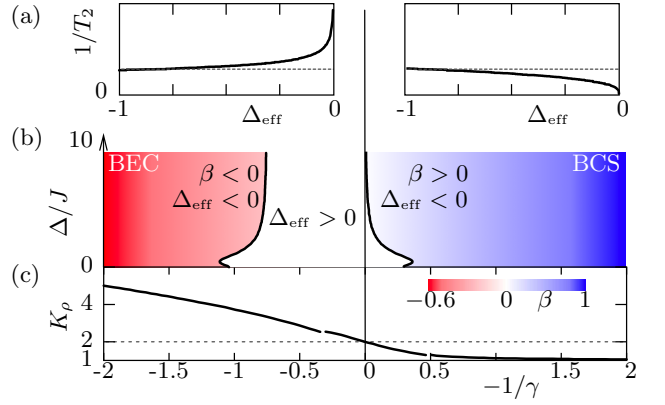


FIG. 2: “Phase diagram” of dephasing rates. (a) Dephasing rate for the coherent Rabi oscillations after a sudden quench to a fixed effective detuning Δ_{eff} (arb. units). For $\gamma < 0$ (BCS) the dephasing rate is suppressed on the effective resonance, while for $\gamma > 0$ (BEC) it diverges. (b) Phase diagram in the plane of $1/\gamma - \Delta/J$. To the left is the BEC regime, where the dephasing rate diverges, and to the right the BCS regime where it is suppressed. The locus of effective resonance ($\Delta_{\text{eff}} = 0$) is marked by the lines enclosing the white area. (c) The Luttinger parameter as a function of $1/\gamma$ [17].

Performing the integral in (9) we obtain the dephasing rate, which is one of our main results

$$1/T_2 \sim \frac{(b_{\text{int}} g \rho_0)^2}{\hbar \epsilon_F} (\Delta_{\text{eff}} / \epsilon_F)^{-1 + \frac{2}{K_{\rho}}}. \quad (12)$$

The most striking aspect of this result is that the point $K_{\rho} = 2$ represents a transition in the nonequilibrium dynamics. While for $K_{\rho} > 2$, the Rabi-oscillations are overdamped, for $K_{\rho} < 2$ the dephasing rate is suppressed as a power-law as $\Delta_{\text{eff}} \rightarrow 0$. That is, the time scale T_2 diverges [see Fig. 2(a)]. The universal behavior of T_2 is generated by an infra-red singularity, or orthogonality catastrophe, due to coupling with critical charge fluctuations in the Luttinger liquid. The effect is analogous to blocking of tunneling through a point contact between one dimensional electron liquids [19]. Scaling analysis, similar to Ref. [19], shows that coupling between the two rotating modes is irrelevant at all orders. Therefore absence of dephasing in the limit $\Delta_{\text{eff}} \rightarrow 0$ is valid beyond the FGR analysis.

All this is strictly true only within our static (time average) approximation. The neglected rapid oscillation of the coupling in the rotating frame will inevitably lead to some dephasing, as will inhomogeneities and finite temperature. In the opposite regime of $K > 2$, the divergence of the dephasing rate signals failure of FGR as $\Delta_{\text{eff}} \rightarrow 0$ and it will be cut off in reality. The practical implications however remain unchanged: overdamped oscillations for $K > 2$ versus strongly suppressed dephasing for $K < 2$ as $\Delta_{\text{eff}} \rightarrow 0$.

It is interesting to contrast the dephasing of oscillations in the interacting quantum liquid with the usual

problem of decoupled two-level systems connected to a reservoir. In both cases “ T_2 ” processes correspond to flipping a two-level system from in-phase to out-of-phase precession. Contrary to the usual case, here the in- and out-of-phase modes are not strictly degenerate. Transferring a particle into the out of phase mode can involve a change in the interaction or axial kinetic energy. The excess energy is absorbed by excitations of the quantum liquid.

In the following we show how a low energy theory with $K_\rho \in [1, \infty]$, $\Delta_s > 0$, and $\Delta_{\text{eff}} \rightarrow 0$ is realized for fermions confined to quasi-one-dimension. We follow the analysis of Fuchs *et al.* [17], who discussed an exactly solvable model relevant to the BCS to BEC cross-over of quasi one-dimensional fermions. The model is parametrized by a dimensionless interaction constant $\gamma = g/(\hbar^2 \rho_0)$, where g is the coupling constant of the Hamiltonian (1). For $1/\gamma \rightarrow -\infty$ the system is in the weakly interacting BCS regime with a spin gap $\Delta_\sigma \propto \exp[-\pi^2/2|\gamma|]$. For $1/\gamma > 0$ the Hamiltonian (1) does not support a bound state. However, this is an artifact of the strictly single channel description. In reality the bound state, and the spin gap go up to the transverse trap frequency as $1/\gamma \rightarrow 0^+$ violating the single channel assumption, this is the so called confinement induced resonance (CIR) [11, 12]. Fuchs *et al.* therefore considered the model (1) supplemented with a bound state at $\epsilon_b = -\infty$ (or $\Delta_\sigma = \infty$) for $\gamma > 0$. We can extract the relevant parameters for the low energy theory from their exact solution. In the crossover regime $1/\gamma \approx 0$, the Luttinger parameter $K_\rho^{-1} \approx (1 - 1/\gamma)/2$ and the chemical potential $\mu_1^{\text{eff}} \approx \epsilon_F(1/4 - 1/3\gamma)$. It is interesting to note that the transition between over-damped Rabi oscillations to suppressed dephasing for $K_\rho < 2$ occurs exactly at the CIR point $1/\gamma = 0$, see Fig. 2(c).

To find the location of the effective resonance $\Delta_{\text{eff}} = 0$, we compute the interaction-induced detuning [17]

$$\Delta_{\text{eff}} = \mu_1^{\text{eff}} - \mu_2^{\text{eff}} \approx \epsilon_F \left[\frac{1}{4} + \frac{1}{3|\gamma|} - |\gamma c_{\text{int}}| \frac{4}{\pi^2} \right]. \quad (13)$$

We see that $\Delta_{\text{eff}} = 0$ requires $1/\gamma \lesssim 1$, which is in the strongly interacting, or cross-over regime. The need for strong interactions is not surprising because the interaction effects have to overcome the Fermi energy of the full mode. In Fig. 2(b) we show the locus of the effective resonance on the two sides of the “transition” in the plane of bare detuning versus interaction parameter. The very different trends on the two sides of the BCS to BEC crossover can be observed even if one does not tune exactly to $\Delta_{\text{eff}} = 0$. The white region in Fig. 2(b) corresponds to $\Delta_{\text{eff}} > 0$ where within our FGR calculation no particles are transferred out of the full mode.

We now turn to briefly discuss a similar setup of spinless bosons with repulsive interactions. As mentioned, this situation is described by Eq. (1) dropping the spin index. The transformation to a rotating frame remains

unchanged. Because of the absence of a Fermi energy the effective detuning is different, and in the Hartree approximation given by $\Delta_{\text{eff}} = \rho_0 g [(1 - |c_{\text{int}}|) - 2|c_{\text{int}}|]$. The low energy theory of the interacting bosons is also a Luttinger liquid. However, in contrast to the fermion case, the single particle tunneling does not vanish on going to the long wavelength limit. In fact it is the most relevant term leading to $1/T_2 \sim \Delta_{\text{eff}}^{-1+\frac{1}{2K}}$. Here, K is the Luttinger parameter controlling the decay of correlations in the filled mode. An orthogonality catastrophe, similar to the Fermions, occurs for $K < 1/2$, which requires bosons with longer range (e.g. dipolar) interactions. Alternatively in the presence of a commensurate lattice potential the transition to a gapped Mott insulator would be accompanied by a transition to suppressed dephasing.

In conclusion, ultracold atoms oscillating between a pair of one dimensional traps following a quantum quench constitute a natural many-body generalization of Rabi oscillations. We described universal behavior in the dephasing rate seen as power-law dependence on an effective detuning parameter. A transition from enhanced to suppressed dephasing at zero detuning is found below a critical value of the Luttinger parameter, which controls the correlation decay exponent in the quantum liquid. The transition is generated by an orthogonality catastrophe in the coupling to critical one dimensional fluctuations. We showed that such a non-equilibrium transition can be observed in a system of attractive fermions in the BCS to BEC cross-over.

We acknowledge stimulating discussions with I. Bloch, S. Trotzky, and E. Demler. This work was supported by DIP, ISF, and the US-Israel BSF.

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